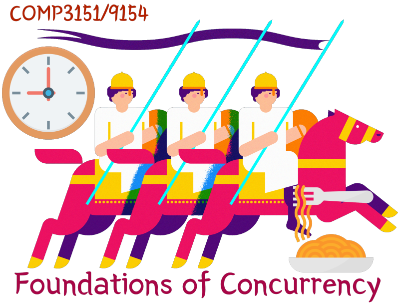

Linear Temporal Logic, Critical Sections and Promela Modelling

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Term 2 2022

Where are we?

Last Lecture

We saw how to treat the semantics of concurrent programs and the properties they should satisfy.

This Lecture

We will give a syntactic way to specify properties (Temporal Logic) and introduce one of two methods we will cover to show properties hold (Model Checking) using the famous Critical Section problem.

Logic

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Example (Propositional Logic Syntax)

- A set of atomic propositions $\mathcal{P} = \{a, b, c, \ldots\}$
- An inductively defined set of formulae:
  - Each $p \in \mathcal{P}$ is a formula.
  - If P and Q are formulae, then $P \wedge Q$ is a formula.
  - If P is a formula, then $\neg P$ is a formula.

(Other connectives are just sugar for these, so we omit them)

Semantics

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Example (Propositional Logic Semantics)

A model for propositional logic is a valuation $\mathcal{V} \subseteq \mathcal{P}$, a set of "true" atomic propositions. We can extend a valuation over an entire formula, giving us a satisfaction relation:

$$\begin{array}{lcl} \mathcal{V} \models p & \Leftrightarrow & p \in \mathcal{V} \\ \mathcal{V} \models \varphi \wedge \psi & \Leftrightarrow & \mathcal{V} \models \varphi \text{ and } \mathcal{V} \models \psi \\ \mathcal{V} \models \neg\varphi & \Leftrightarrow & \mathcal{V} \not\models \varphi \end{array}$$

We read $\mathcal{V} \models \varphi$ as $\mathcal{V}$ "satisfies" φ.

LTL

Linear temporal logic (LTL) is a *logic* designed to describe linear time properties.

Linear temporal logic syntax

We have normal propositional operators:

- $p \in \mathcal{P}$ is an LTL formula.
- If φ, ψ are LTL formulae, then $\varphi \wedge \psi$ is an LTL formula.
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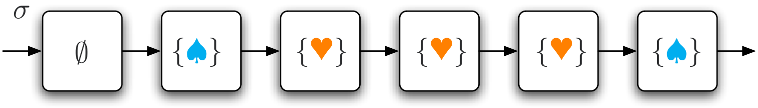
We also have modal or temporal operators:

- If φ is an LTL formula, then $\bigcirc\ \varphi$ is an LTL formula.
- If φ, ψ are LTL formulae, then $\varphi\ \mathcal{U}\ \psi$ is an LTL formula.

LTL Semantics in Pictures



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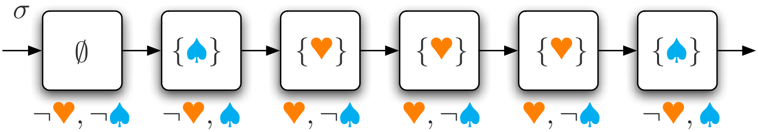

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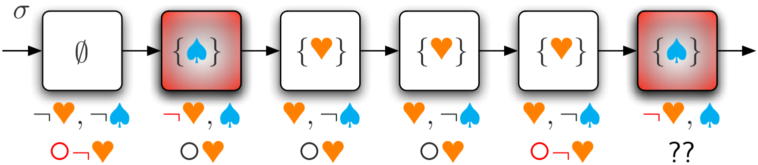

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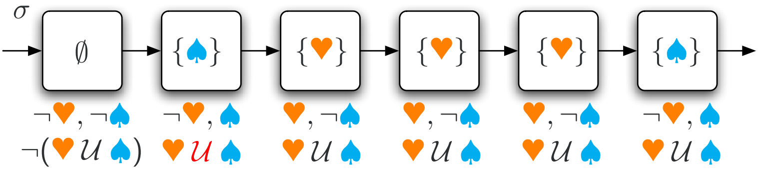

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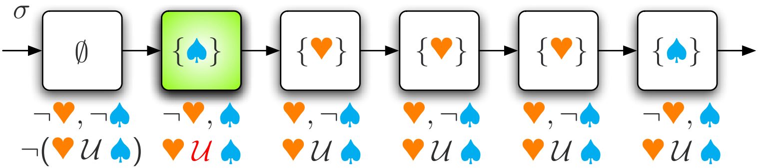

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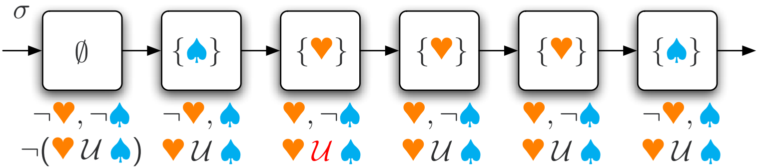

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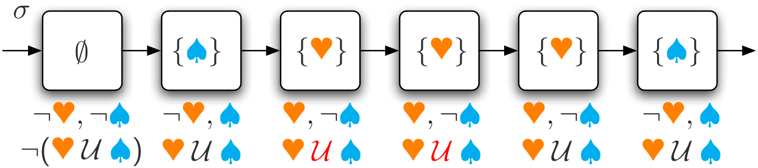

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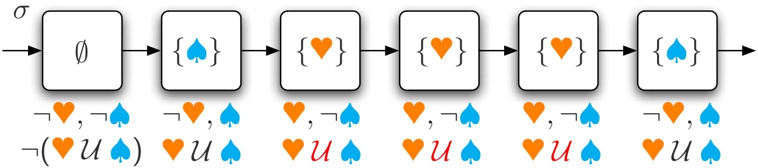

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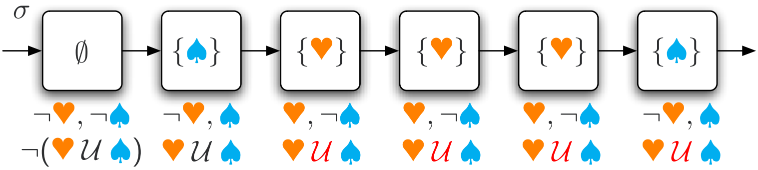

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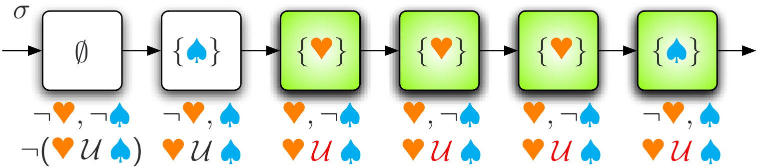

LTL Semantics

Let $\sigma = \sigma_0\sigma_1\sigma_2\sigma_3\sigma_4\sigma_5\ldots$ be a behaviour. Then define notation:

- $\sigma|_0 = \sigma$
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$$\sigma \models p \qquad \Leftrightarrow \quad p \in \sigma_0$$

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\sigma \models \varphi\ \mathcal{U}\ \psi & \Leftrightarrow & \text{There exists an } i \text{ such that } \sigma|_i \models \psi \\
 & & \text{and for all } j < i,\ \sigma|_j \models \varphi
\end{array}
$$

We say $P \models \varphi$ iff $\forall \sigma \in [\![P]\!].\ \sigma \models \varphi$.

Derived Operators

The operator $\Diamond\ \varphi$ ("finally" or "eventually") says that φ will be true at some point.

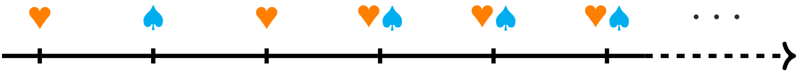
The operator $\Box\ \varphi$ ("globally" or "always") says that φ is always true from now on.

Exercise

- Give the semantics of $\Box$ and $\Diamond$.
- Define $\Box$ and $\Diamond$ in terms of other operators.

More Exercises

Let ρ be this behaviour:



$\rho \models$ ♥?

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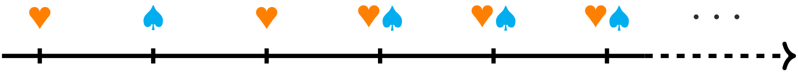

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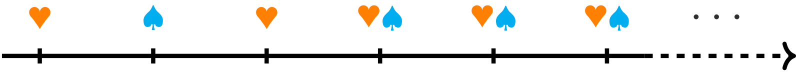

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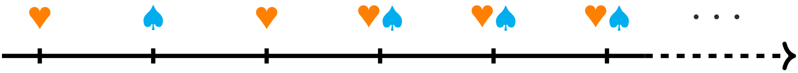

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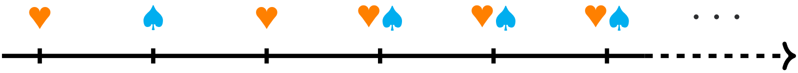

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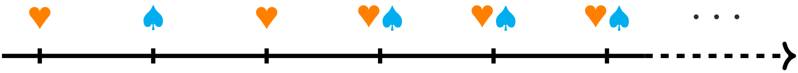

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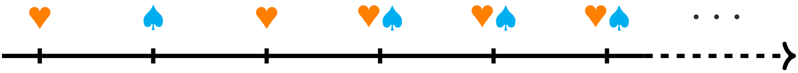

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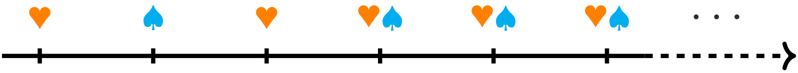

More Derived Operators

- Define "Infinitely Often" in LTL.
- Define "Almost Globally" in LTL (always true from some point onwards).

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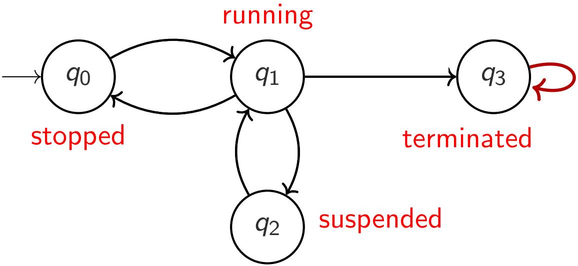

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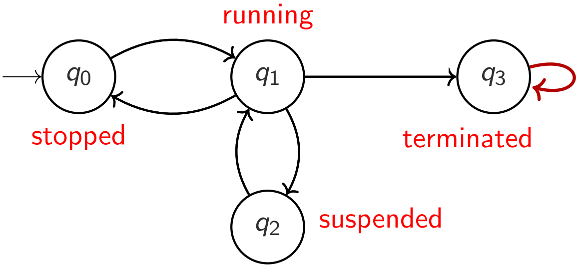

We can see that it is always possible for a run to move to the terminated state. How do we express this in LTL? **We can't!** — it is a *branching time* property.

Branching Time

Dealing with branching time properties requires a different logic called CTL (Computation Tree Logic). Learn about it in COMP3153/9153 or COMP6752.

A counting argument for mechanical aids

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3	20	1680	$2^{18.4}$	$2^{27.3}$	$2^{36.9}$
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So, for 6 processes consisting of 6 sequential atomic actions each, that's merely 2 670 177 736 637 149 247 308 800 scenarios.

Do come back when you're done testing!

Sobering Conclusion

For any realistic concurrent program, it is *infeasible to test* all possible scenarios.

We need to apply smarter techniques than brute-force testing to establish properties of concurrent programs.
Formal methods let us reason about programs, or, if that is too hard, about *abstractions* of programs.

Industrially applicable formal methods

To verify that program P has property φ (i.e. $P \models \varphi$), we can use:

- *model checking* — exhaustively searching through (an efficient representation of) P's state space to find a *counterexample* to φ
- *theorem proving* — construct a (formal) proof of φ

To be relevant in practice, these techniques must be supported by *tools*.

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Pros: easy to use push-button technology; instructive counter examples (error traces) help debugging

Cons: *state (space) explosion problem*

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Answer

COMP3153/9153 *Algorithmic Verification* (should run in T2)

(Interactive) Theorem Proving

Pros: no (theoretical) limits on state spaces

Cons: *requires* expert users (e.g. *skilled computer scientists*, mathematicians, or logicians) to hand-crank through proofs

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COMP4161 *Advanced Verification* (should run in T3)

SPIN

A model checker for concurrent systems with a lot of useful features and support for LTL model checking.

`http://www.spinroot.com`

Programs are modelled in the Promela language.

Promela in brief

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Warning

Variables of non-fixed size like `int` are of machine determined size, like C.

Example 1: Hello World

Johannes will demonstrate the basics of `proctype` and `run` using some simple examples.

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Take-away

You can use SPIN to *randomly simulate* Promela programs as well as model check them.

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Johannes will demonstrate a program that exhibits non-deterministic behaviour due to scheduling.

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You can also add explicit non-determinism using `if` and `do` blocks:

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if
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:: (n >= 0) -> n = n - 2;
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What would happen without the `else` line?

Guards

The arrows in the previous slide are just sugar for semicolons:

```
if
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:: (n >= 0); n = n - 2;
:: (n % 3 == 0); n = 3;
fi
```

A boolean expression by itself forms a *guard*. Execution can only progress past a guard if the boolean expression evaluates to true (non-zero).
If the entire system cannot make progress, that is called deadlock.
SPIN can detect deadlock in Promela programs.

mtype and Looping

```
mtype = {RED, YELLOW, GREEN};
active proctype TrafficLight() {
    mtype state = GREEN;
    do
    :: (state == GREEN) -> state = YELLOW;
    :: (state == YELLOW) -> state = RED;
    :: (state == RED) -> state = GREEN;
    od
}
```

Non-determinism can be avoided by making guards mutually exclusive. Exit loops with `break`.

Volatile Variables

var $y, z \leftarrow 0, 0$	
p_1: **var** x;	q_1: $y \leftarrow 1$;
p_2: $x \leftarrow y + z$;	q_2: $z \leftarrow 2$;

Question

What are the possible final values of x?

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It is possible, as we cannot guarantee that the statement p_2 is executed atomically — that is, as one step.

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Typically, we require that each statement only accesses (reads from or writes to) at most one shared variable at a time. Otherwise, we cannot guarantee that each statement is one atomic step. This is called the *limited critical reference* restriction.

Ensuring Atomicity

We will often have multiple actions that we wish to group into one step, i.e. to execute atomically.

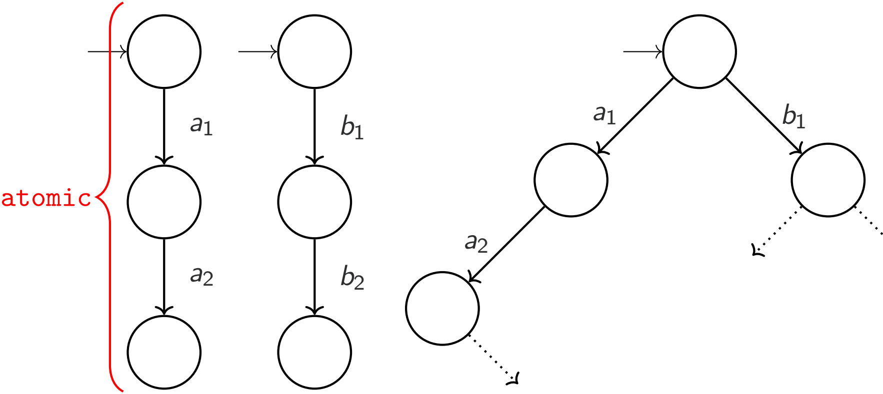
Example (Counters)

In our counter example, if each process executes the loop body atomically the result number can be guaranteed.

In Promela we can simply state this requirement, but in real programming languages we must use synchronisation techniques to achieve this.

`atomic` and `d_step`

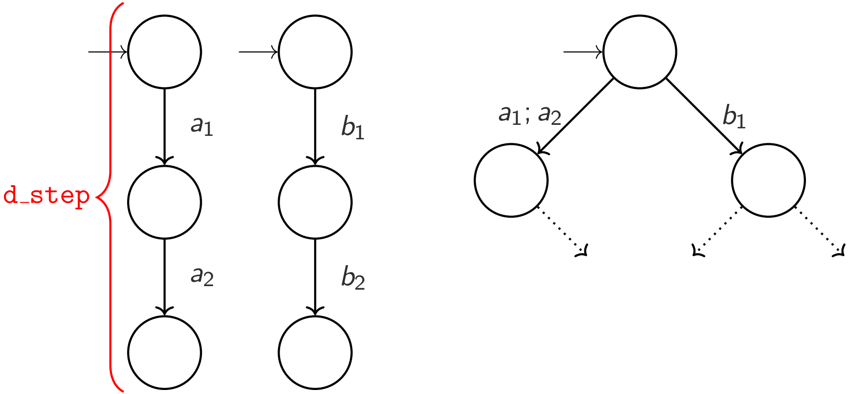
Grouping statements in Promela with `atomic` prevents them from being interrupted.



If a statement in an atomic block is blocked, atomicity is temporarily suspended and another process may run.

`atomic` and `d_step`

Grouping statements with `d_step` is more efficient than `atomic`, as it groups them all into **one transition**.



Non-determinism (`if`,`do`) is not allowed in `d_step`. If a statement in the block blocks, a runtime error is raised.

Atomicity

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A sketch of the problem can be outlined as follows:

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The non-critical section models the possibility that a process may do something else. It can take any amount of time (even infinite).

Our task is to find a pre- and post-protocol such that certain atomicity properties are satisfied.

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We want to ensure two main properties and two secondary ones:

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Question

Which is safety and which is liveness?

Eventual Entry is liveness, the rest are safety.

First Attempt

We can implement **await** using primitive machine instructions or OS syscalls, or even using a busy-waiting loop.

var *turn* $\leftarrow 1$	
forever do	**forever do**
p_1 *non-critical section*	q_1 *non-critical section*
p_2 **await** $turn = 1;$	q_2 **await** $turn = 2;$
p_3 **critical section**	q_3 **critical section**
p_4 $turn \leftarrow 2$	q_4 $turn \leftarrow 1$

Question

Mutual Exclusion?

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forever do	**forever do**
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p_3 **critical section**	q_3 **critical section**
p_4 $turn \leftarrow 2$	q_4 $turn \leftarrow 1$

Question

Mutual Exclusion? Yup!

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p_4 $turn \leftarrow 2$	q_4 $turn \leftarrow 1$

Question

Mutual Exclusion? Yup!
Other criteria?

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Question

Mutual Exclusion? Yup!
Other criteria? Nope!

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p_3 **critical section**	q_3 **critical section**
p_4 $turn \leftarrow 2$	q_4 $turn \leftarrow 1$

Question

Mutual Exclusion? Yup!
Other criteria? Nope! What if q_1 never finishes?

Second Attempt

var *wantp*, *wantq* $\leftarrow$ False, False	
forever do	**forever do**
p_1 *non-critical section*	q_1 *non-critical section*
p_2 **await** *wantq* = False;	q_2 **await** *wantp* = False;
p_3 *wantp* $\leftarrow$ True;	q_3 *wantq* $\leftarrow$ True;
p_4 **critical section**	q_4 **critical section**
p_7 *wantp* $\leftarrow$ False	q_7 *wantq* $\leftarrow$ False

Second Attempt

var *wantp*, *wantq* ← False, False	
forever do	**forever do**
p_1 *non-critical section*	q_1 *non-critical section*
p_2 **await** *wantq* = False;	q_2 **await** *wantp* = False;
p_3 *wantp* ← True;	q_3 *wantq* ← True;
p_4 **critical section**	q_4 **critical section**
p_7 *wantp* ← False	q_7 *wantq* ← False

Mutual exclusion is violated if they execute in lock-step (i.e. $p_1q_1p_2q_2p_3q_3$ etc.)

Third Attempt

var *wantp*, *wantq* ← False, False	
forever do	**forever do**
p_1 *non-critical section*	q_1 *non-critical section*
p_2 *wantp* ← True;	q_2 *wantq* ← True;
p_3 **await** *wantq* = False;	q_3 **await** *wantp* = False;
p_4 **critical section**	q_4 **critical section**
p_7 *wantp* ← False	q_7 *wantq* ← False

Third Attempt

var *wantp*, *wantq* ← False, False	
forever do	**forever do**
p_1 *non-critical section*	q_1 *non-critical section*
p_2 $\mathit{wantp} \leftarrow \mathrm{True}$;	q_2 $\mathit{wantq} \leftarrow \mathrm{True}$;
p_3 **await** $\mathit{wantq} = \mathrm{False}$;	q_3 **await** $\mathit{wantp} = \mathrm{False}$;
p_4 **critical section**	q_4 **critical section**
p_7 $\mathit{wantp} \leftarrow \mathrm{False}$	q_7 $\mathit{wantq} \leftarrow \mathrm{False}$

Now we have a deadlock (or stuck state) if they proceed in lock step.

Fourth Attempt

var *wantp*, *wantq* ← False, False	
forever do	**forever do**
p_1 *non-critical section*	q_1 *non-critical section*
p_2 *wantp* ← True;	q_2 *wantq* ← True;
p_3 **while** *wantq* **do**	q_3 **while** *wantp* **do**
p_4 *wantp* ← False;	q_4 *wantq* ← False;
p_5 *wantp* ← True	q_5 *wantq* ← True
p_6 **critical section**	q_6 **critical section**
p_7 *wantp* ← False	q_7 *wantq* ← False

Fourth Attempt

var $wantp, wantq \leftarrow$ False, False	
forever do	**forever do**
p_1 *non-critical section*	q_1 *non-critical section*
p_2 $wantp \leftarrow$ True;	q_2 $wantq \leftarrow$ True;
p_3 **while** $wantq$ **do**	q_3 **while** $wantp$ **do**
p_4 $wantp \leftarrow$ False;	q_4 $wantq \leftarrow$ False;
p_5 $wantp \leftarrow$ True	q_5 $wantq \leftarrow$ True
p_6 **critical section**	q_6 **critical section**
p_7 $wantp \leftarrow$ False	q_7 $wantq \leftarrow$ False

We have replaced the deadlock with live lock (looping) if they continuously proceed in lock-step.

Fifth Attempt

var *wantp*, *wantq* ← False, False var *turn* ← 1	
forever do	**forever do**
p_1 *non-critical section*	q_1 *non-critical section*
p_2 *wantp* = True;	q_2 *wantq* = True;
p_3 **while** *wantq* **do**	q_3 **while** *wantp* **do**
p_4 **if** *turn* = 2 **then**	q_4 **if** *turn* = 1 **then**
p_5 *wantp* ← False;	q_5 *wantq* ← False;
p_6 **await** *turn* = 1;	q_6 **await** *turn* = 2;
p_7 *wantp* ← True	q_7 *wantq* ← True
p_8 **critical section**	q_8 **critical section**
p_9 *turn* ← 2	q_9 *turn* ← 1
p_{10} *wantp* ← False	q_{10} *wantq* ← False

Reviewing this attempt

The fifth attempt (Dekker's algorithm) works well except if the scheduler pathologically tries to run the loop at $q_3 \cdots q_7$ when $turn = 2$ over and over rather than run the process p (or vice versa).

What would we need to assume to prevent this?

Reviewing this attempt

The fifth attempt (Dekker's algorithm) works well except if the scheduler pathologically tries to run the loop at $q_3 \cdots q_7$ when $turn = 2$ over and over rather than run the process p (or vice versa).
What would we need to assume to prevent this?

Fairness

The *fairness assumption* means that if a process can always make a move, it will eventually be scheduled to make that move.

With this assumption, Dekker's algorithm is correct.

Expressing Fairness in LTL

Let $\mathsf{enabled}(\pi)$ and $\mathsf{taken}(\pi)$ be predicates true in a state iff an action π is enabled, resp., taken.

Examples

Weak fairness for action π is then expressible as:

$$\Box(\Box\mathsf{enabled}(\pi) \Rightarrow \Diamond\mathsf{taken}(\pi))$$

Strong fairness for action π is then expressible as:

$$\Box(\Box\Diamond\mathsf{enabled}(\pi) \Rightarrow \Diamond\mathsf{taken}(\pi))$$

Promela can assume weak fairness when checking models.

What now?

- Do the homework exercises and submit them before the Friday lecture.
- Assignment 0 (warm-up) will be out in W2. You have enough knowledge to start it, but not yet enough to finish it.
- Get spin (and ispin) working on your development environment (or use VLAB/ssh)